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ABSTRACT

This paper reviews literature related to two questions: (1) How do preschoolers acquire numerical concepts? (2) What range of basic numerical knowledge do beginning kindergarteners possess? The first section briefly highlights main ideas of learning theories. The second section discusses Piaget's concept of conservation of number and how young prekindergarten children learn from their environment. The third section includes summaries of research on the mathematical knowledge of entering kindergarteners. Early number skills studied by researchers include counting by rote, matching, comparing, naming, adding to and taking away, and relational judging. It is emphasized that preschoolers have a substantial knowledge of numerical concepts. (RH)

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Numerical Acquisition in Early Childhood Years

Pamela Lam

Introduction

Numeration is the heart of contemporary mathematics education" (Maertens, Waite, & Jones, 1977, p. 181). According to Piaget, number is an example of "logico-mathematical knowledge, which is an aspect of knowledge that is different from both physical and social (arbitrary) knowledge" (Kamii, 1976, p. 5). Number presents a special challenge to children, since they are not physical objects that can be manipulated. Numerosity is an abstract knowledge for everybody. Children encounter problem situations that require quantitative solutions and number responses in their daily lives. It is the fact that number is an integral part of our daily lives which ensures that children need to have a clear understanding of numerical concepts.

Most of the mathematical programs that are offered in schools have the purpose of assisting children to move from the perceptual, intuitive thinking to operational and logical thinking. Through developing a logical way of thinking, children learn to understand better the numerical concepts. In The Child's Conception of Number, Jean Piaget had stated that "the construction of number goes hand-in-hand with the development of logic and that a pre-numerical period corresponds to the pre-logical level" (Piaget, 1965, p. viii). However, it is a great mistake to suppose that a child

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acquires the notion of number just from teaching. On the contrary, a child develops concepts independently and spontaneously by himself. True understanding of the concepts can only come from the child's own maturation and mental growth (Stendler, 1964). The details of specific computation can be taught only after the child himself has constructed the logic underlying the addition of two numbers.

Statement of the Problem

When an infant is born, he does not know any mathematics. However, by the time that the child enters kindergarten, he has encountered problem situations that require him to use quantitative solutions, and it is assumed that he has possessed certain basic mathematical skills and knowledge. Some time in between the infant years to kindergarten, a child has acquired certain fundamental mathematical concepts without any formal training. This becomes a point of curiosity and doubt to most people. Therefore, the crux of this paper is to review related literature on: (1) How do preschoolers acquire numerical concepts? (2) What range of basic numerical knowledge do beginning kindergarteners possess? Since mathematical concepts is such a broad field, it is almost impossible to include every topic within the scope of this paper. Hence for the sake of simplicity, the area in this paper is limited to early numerical concepts, which is considered the basic foundation in mathematical learning.

Review of Related Research and Literature

General Overview of Learning Theories

Learning is thought of generally as "a relatively permanent change in behavior that results from the interaction (experiences) of an individual with his environment" (Thornburg, 1973, p. 4). There are wide individual differences existing in the abilities of children to learn and to solve problems, and these differences are complex and difficult to determine.

In his theory, Piaget postulates definite stages of intellectual development through which each child must pass, and he states approximate ages at which children tend to reach them. The four main stages of intellectual development described by Piaget are: (1) Sensory-motor stage--this stage encompasses the first eighteen months of the child. The infant comes to know the world through the patterns of action he applies to it. At the end of this stage, the child would construct the sense of permanence of objects. (2) Preoperational stage--this stage lasts until about seven years of age. In this stage, the child is limited to judgement which is in terms of perceptions. He is not yet engaged in logical thought. His logic is not systematic, and is better described as 'semilogic'. He lacks an understanding of hierarchical classifications. He cannot yet readily reverse his thinking or deal with reciprocal relationship. (3) Stage of concrete operations--this stage lasts from seven to thirteen years of age. The child begins to think more logically and systematically, and is more amenable to instruction. (4) Stage of

formal operations--this stage begins at about age thirteen. The child begins to solve problems with abstract reasoning, instead of mere concrete operations (Almy, 1975, p. 62-64). It becomes obvious that in this review, we are focused in the second stage of intellectual development--the preoperational stage.

According to the theory of Jean Piaget, there are four factors that are involved in the transition from one stage of development to the next. These factors include "maturation, the child's actions on his physical environment, social transmission, including both the tuition of adults and collaboration with other children, and the processes of self-regulation or equilibration" (Almy, 1966, p. v, vi). Stages, by themselves, are only descriptive and explain nothing. However, they do provide the teacher with some notion of what to expect from children of those ages (Almy, 1975).

Learning-environmentalists agree with stage-dependent theorists that behavior changes relate in some ways to chronological age. However, they disagree as to the cause of that relationship, as well as to the emphasis placed on it. One learning-environmentalist, Robert Gagne suggested that "the child progresses from one point to the next in his development... because he learns an ordered set of capabilities which build on each other in progressive fashion" (Gibson, 1978, p. 74).

The review of literature on learning theories has probably cast more doubt and confusion than clarification, because different developmental theorists have their own individual principles of learning. Since we already have some basic ideas about the general theories of learning, maybe we should focus back to our original interest of mathematics learning for young children. How does a young child acquire his mathematical concepts?

Mathematical Learning for Young Children

In reviewing the literature for this subject, one would probably realize that there is no way that you can omit Piaget's contribution to children's learning of number concept, based on the acquisition of conservation. The topic on which most research has been done is conservation. Conservation is defined as "a child's realization of the principle that a particular dimension of an object (or set) may remain invariant under changes in other, irrelevant aspects of the situation" (Wohlwill & Lowe, 1962, p. 153). Conservation of number involves "a child's understanding of the fact that the number of objects in a set remains the same despite all changes in perceptual configuration. The number of objects in a set changes only under the operations of putting into a set and taking out of a set" (Levin, 1969, p. 3).

> The ability to conserve is revealed when the child grasps the mathematical idea that number is not changed when a set of objects is partitioned into subgroups, and the physical idea that mass or substance does not change when the shape or appearance

of an object is transformed (Almy et. al., 1966, p.9)

Piaget postulated that the beginning of conservation starts somewhere in the preoperational stage (Almy, 1975). The development of conservation can be broken into three stages. Stage one represents a lack of conservation, stage two represents a transition between nonconservation and conservation, while stage three represents complete conservation. In his conservation task experiment in dealing with elementary numbers, Piaget described that in the non-conservation stage, the child cannot make an equivalent set, because the child only base his or her judgement on the spatial frontiers of the row. In this transition stage, the child can make an equivalent set, but cannot conserve the equivalence. In other words, the child can make another row of objects with the same number of objects as given in the original task, but if the spatial arrangement of the two rows is changed, either by lengthening or shortening the spatial frontiers, the child will be confused about the equivalence of the two sets, because his conservational level has not been well developed yet. In the complete conservation stage, the child encounters no problem in equivalency. When the child was asked how he or she knows that there are just as many in one row as in the other, the child usually gives one of the following explanations:

(1) identity--You didn't take away or add anything. All you did was move the things.
(2) reversibility--If you move the things back to the way they were before, you'll see that there is the same amount.

(3) compensation--This line is longer, but the things are spread out more. (Kamii, 1976, p. 3)

Kindergarten children, according to Piaget, are at the transition stage between nonconservation and conservation (Levin, 1969). Stendler comments that the basic tenet is that "children must grasp the principle of conservation before they can develop the concept of number" (Stendler, 1964, p.334). It is noted that children who conserve at an early age do better in beginning arithmetic than those who are late in acquiring conservation (Almy et. al., 1966). Conservation of number, according to Copeland, may be achieved by a bright 5-year-old, but in the typical situation, it probably takes about a 7-year-old to accomplish the task (Copeland, 1974). Does this conclude that we should not introduce any number concepts to children before they reach the age of seven? NO!

One of the causes of controversy about Piaget's theory is the question of readiness. Piaget stresses the fact that a child must reach the operational level of thinking before he can be expected to understand the concept of number. There is an age limit below which children, owing to immaturity, may not be able to develop any understanding of a particular concept (Gibson, 1978). This seems to be contested in present literature by Bruner's theory of readiness stating that anything can be taught to a child at any age in some intellectually honest form if certain restrictions or agreements can be met (Bruner, 1966).

Piaget's view of children's intellectual development has laid a foundation for the understanding of the mathematical

learning in young children. Even though his ideas seems very controversial, at least we can gain some benefit from it. One of the most important factor is that we can likely assume that the child is at the preoperational stage between two and seven years of age. Because of the absence of abstract reasoning, a child at this stage can only deal with concrete knowledge. The teacher should therefore provide a learning situation that will provoke the desired learning for the child by using concrete materials.

The next point is to discuss how the young children learn from his environment before they enter kindergarten for formal training under teachers. It is a great mistake to suppose that a child acquires the notion of number and other mathematical concepts only from teachers. On the contrary, a child develops many number concepts himself through observation of daily activities around him. The numerosity of children's toys and games, play experience, television and general life environment have vastly increased the knowledge of children in this generation (Rea & Reys, 1970). Children begin to acquire mathematical ideas almost from the time they became aware of the world around them (Suydam & Weaver, 1975).

In Piaget's theory, children learn numbers through reflective abstraction or constructive abstraction, focusing on certain physical properties. However, a child begins abstracting from physical properties of a small set. Later, he can then learn to involve the creation of mental relationships between or among objects. This is especially true when

the child learns large numbers like 999 or 1000. It is impossible to learn every number all the way to infinity by abstraction from sets of objects. Therefore, the child has to learn to abstract by constructing relationships among numbers (Kamii, 1976). However, children learn many things about the physical world in which they live before they abstract the idea of number. "Through manipulating a set of objects and gradually, through much experience, children can form in their mind a mathematical idea or model that is abstract " (Gibb & Castaneda, 1975, p.97). Readiness for the systematic development of the concept of number can be provided by developing the child's ability to perceive a collection of objects as an entity (Gibb & Casteneda, 1975).

Children very probably begin their number and quantitative notions with comparisons of size. Children use perceptual comparisons, and this can be detected by observing the vocabulary that children use to express quantitative ideas, such as 'same', 'more', 'just as many', 'not as many', 'lower', 'higher', 'bigger', and 'smaller', which arises naturally in the comparison situations (Payne & Rathmell, 1975). Piaget suggests that children first consider comparing quantity in 'global' terms. They talk about sets as having 'a lot' of elements rather than five or six, and they describe operations as adding 'some' rather than specifically two or three elements. As the number knowledge develop, the children probably refine the global groupings into unit quantities (Brush, 1978).

Another prenumber knowledge that children before five have acquired is classifying. Very young children can classify, and they put together things that are alike or that belong together. They may or may not be able to label what they have classified. Classifying is a part of everyday life and is probably among the many activities offered in the preschool program (Gibb & Castaneda, 1975).

Ordering or seriation builds on the knowledge of comparing. For the young children, ordering involves a physical arrangement of objects or sets of objects. Obviously children order on the basis of known attributes that they can compare, e.g. length, size, etc. (Gibb & Castaneda, 1975). A child is dealing with number in an ordinal sense when he is seriating. Ordinality deals with order or succession, and it answers the question of 'which one?'. Piaget indicates that children in the preoperational stage use trial-and-error method when they perform the seriation task (Piaget, 1965).

Children usually start counting at a very young age. It is mentioned by Piaget that "the ability to count is no guarantee that the child has a stable concept of number" (Lovell, 1961, p. 45). Furthermore, a study done by Dodwell confirms Piaget's contention that young children do not fully understand the number concept even though they may be able to count (Dodwell, 1960). Copeland mentions that there are three levels of ability in counting. Stage one represents only rote counting. In this stage, children only memorize a sequence of sound. It is purely rote type or memory

learning. The child has not yet learned to establish one-to-one correspondence by matching number names to the objects being counted. Obviously the child still determines the quantity of sets in a global, perceptual way. Stage two represents rational counting in which the child deals with one-to-one correspondence or matching, and he learns to match the number names or numerals to objects. The concept of number may not be completely understandable to the children, but they count correctly. In the final stage, the children have already mastered the conservation of number and is therefore counting with lasting equivalence (Copeland, 1974).

It is difficult for a 5-year-old to establish strong images of numbers that are more than five, because the sets are nonelemental groupings. Children and adults tend to group and regroup objects in order to perceive them (Maertens et. al., 1977). Freeman (1911) tested both adults and children and found that an average person can perceive at most five objects. Studies prove that elementally grouped objects (sets of less than five objects aligned or stacked in orderly fashion) are more easily perceived by the children than the nonelementally grouped objects, and that uniform patterns of elemental groupings are more easily perceived than nonuniform patterns of elementally grouped set of objects (Maertens et. al., 1977)

Very little is known from research about how children acquire mathematical learning in the preschool years, and a review of literature provided very few clues in this area.

Generally, studies are focused on older children, probably because it is easier to obtain results from children who can verbalize well and explain their actions.

It is also important for us to find out how much children already know when they enter kindergarten. If we are aware of what the children already know, we can probably achieve a meaningful starting point for them. Before an effective mathematics program can be introduced at the kindergarten level, it is imperative to know about the children's level of mathematical readiness at the time they enter school.

Mathematical Knowledge of Entering Kindergarteners

It is clear that children have acquired many mathematical ideas before they begin formal instruction in school. The variability of methods used in assessing the knowledge of children entering kindergarten makes it difficult to affirm precisely what children in general know about number concept. Different studies have revealed some similar and some different results. A summary of some of these studies may provide an idea of what number knowledge a 5-year-old possesses before he starts school.

Rea and Reys (1971) had done a cross-sectional study of 727 urban children who had just en-tered kindergarten. Some of the results for the study indicated that: (1) Very few children could orally name the number in a set of above 20. (2) Seventy-five percent of the children knew what comes after 3, but only 43 percent of the children knew what comes before 3. Part of the reasons may be due to the fact that

some children only rote count at this age without really understanding the numerousness of the number concept. (3) Cardinal knowledge was very well established for this group of children, but ordinality was at a much lower level. (4) Children in the group could compare very well.

Another part of the study which was done in an earlier time by Rea and Reys (1970) indicated that almost 40 percent of their entering kindergarten group could count by rote to 20, and about the same percent could count rationally to 20. More than 75 percent could count by rote beyond 10 and about the same percent of children could count rationally beyond 10. Over 50 percent of the children correctly counted beyond 14.

A big study was done in six elementary school districts in Southern California, using a sample of 595 kindergarten entrants. The results indicated that: (1) Seventy-five percent of the sample children could identify three dots, and marked three dots in a larger group of dots. They could also compare very well, and could pick out the longest pencil or identify the highest degree in the thermometer readings. (2) Only 50 percent could identify the numerousness of 5 and 9, and read the numeral 4. (3) Only 50 percent of the sample children could pick out the shortest length of objects. (4) Even fewer children (25 percent) could mark seven, nine and fourteen dots from a group, read numerals 6, 9 and 0, understood the cardinal meaning of 2 and 5, located the second or the in-between objects in a series, recognized half of a

whole, distinguished 'fewest' from groups of objects, or use one-to-one correspondence to match elements of equivalent sets. (5) Fewer than 25 percent of the children read the numeral 23, or wrote the numeral 4 and 8, located the fifth element in a series, knew the sum of 2 and 3, or 2 and 1. However, the results of this study revealed that as the chronological age increases, the mathematical achievement of the subject also increases (Williams, 1965).

Bjonerud (1960) had done a study with 127 beginning kindergarteners in Michigan. The results of the study indicated some similarities with the other studies. It was found that: (1) The mean counting by rote for that particular group was 19. There were five children who could really count to 100 or more. For some children, the counting was not all that accurate. As the higher numbers were reached, a few counted in blocks of sequence, jumping several numbers and then continued on some without errors. (2) Less than 10 percent of the children really understood a sequence of odd numbers, but approximately 20 percent succeeded when a sequence of even numbers less than 10 were used. (3) More than half of the children had some understanding of the ordinal number concept. Approximately 95 percent understood the concept of 'first', more than 70 percent knew 'middle' and 'last'. At least 50 percent knew 'second' and 'fourth'. (4) Almost all the children in the study possessed considerable understanding of number selection skills and were able to recognize a quantity of items numbering less than four

immediately. Some were able to recognize more than four items, but less than nine items. All of the children were able to select quantities of three or less.

Recently, Kraner (1977) administered a quantitative skill/concept test to 273 children between the ages of 3 to 6. The purpose of his study was to investigate the acquisition age of quantitative concepts of young children within that age range. The results of his study revealed that: (1) Both rote and rational counting by ones to 20 were mastered by children of 5½ years old. (2) As early as 3½ years old, children could count 1 to 3, and counted objects to 2. Serial counting was not achieved until about 6 years old. (3) The more general concepts of 'all' and 'empty' were acquired earlier (about 3½ years old) than the more definitive concepts of 'more than' and 'less than'. The concept of 'more than' was achieved earlier than 'less than' for most of the children. (4) 'First' was the earliest emerging concept, but 'beginning' was the latest emerging concept. The significance for this study was that it brings out some interesting general conclusions in this area of interest. Among them were: (1) There appeared to be a common sequence and age in the acquisition of quantitative concepts. (2) Several quantitative concepts commonly considered to be acquired by entering first-grade students and required for successful first grade mathematics achievement had not been mastered by a large percentage of children at 6½ years of age in this study. (3) When compared to some studies of

five to ten years ago, the acquisition age of quantitative concepts was occurring at an earlier age.

Some research has gone far below the kindergarten level to investigate the preschool children's knowledge of numerical concepts. Johnston (1964) analyzed the vocabulary of 4- and 5-year-olds and found that the number words comprised approximately 50 percent of all of their mathematical vocabulary. A recent study by Brush (1978) was done to investigate preschool children's knowledge of addition and subtraction in the sense of physical manipulation. The study was conducted in two parts. The sample for the first part included 26 children ranging in age from 3 years 11 months to 5 years 1 month. The operations of addition and subtraction was presented to them as 'adding to' and 'taking away', changing the amounts in a set. The results suggested that: (1) Young children in the study performed well on the addition and subtraction problems. As the tasks became more complex, more children correctly assessed addition tasks than their subtraction counterparts. This interesting result may be accounted to the fact that children have encountered the creation of 'more' more frequently than 'less' in their home environment, and that they have got used to counting numbers forward more often than backward.

The sample for the second part of the study included more older children. There were 60 children, ranging in ages from 4 years to 6 years 11 months. This part of the study confirmed

the results that preschool children appeared to be knowledgeable about the operations of addition and subtraction even before they had received any formal training in them. Tests with the older children indicated that they could all perform some addition and subtraction tasks, but fewer children were correct on the subtraction tasks than on the addition. Both parts of the study showed that most preschool children had a quite sophisticated knowledge of the elementary operations and thus implied that a direct teaching of these ideas may not be necessary. Children discovered the nature of addition and subtraction operation outside of their formal education, usually prior to entering first grade (Brush, 1978).

It is probably true that before children are taught to add or subtract in a formal sense, they have picked up some of the properties of these operations. Although they may not be interpreting the physical manipulation in a numerical sense, they do recognize the general effect of the manipulations.

Bullock and Gelman have done a study on some young children, ranging in ages from 2 years 5 months to 4 years 11 months. The children were asked to compare two number pairs on the basis of a common ordering relation. The results of the study revealed that children as young as 2½ years of age could make number-based relational judgments (Bullock & Gelman, 1977).

As for the knowledge in relative numbers, different

studies reveal dissimilar results. Donaldson and Balfour (1968) reported that 3-year-olds could not differentiate 'less' from 'more', and he postulated that in these children, the meaning of 'more' had probably been overextended to include the meaning of 'less' as well. However Bryant (1974) maintained that children initially adopted relative codes for numbers, such as 'more' and 'less'. Therefore a 4-year-old should have no difficulty with relative number tasks.

From all the above mentioned studies, we can see that the extent and nature of mathematical achievement of kindergarten entrants are far-ranging. Some results revealed that the different mathematical achievement level are associated with a number of factors. Among them are language, home environment and experiences around the children.

In the experiment done by Jean Tough to investigate the contribution made by relevant experience and language to the formation of a number concept by 5-year-old children, she concluded that the group with whom the appropriate language was introduced at the same time as the experience made significant progress in the learning of the concept under consideration. Tough commented that the basic concept was more efficiently formed when language and concrete experience appeared together (Tough, 1966).

The home environment is in some way affecting the learning of the child. In Williams' study, the results revealed the probability of positive correlation between the child's mathematical achievement and the parent's

occupation. Children whose parents' occupation placed them in higher socioeconomic classes made significantly higher scores on the mathematics test than did those whose parents' occupations were categorized as lower socioeconomic status. Some of the circumstances or learning conditions existing in the homes apparently influence some kindergarten entrants to attain a higher level of mathematical achievement than other of equal mental ability (Williams, 1965). The environment and experiences that the children have around them can greatly influence their learning. Appropriate mathematical instruction in terms of concrete materials, would be beneficial and profitable to any child (Williams, 1965).

Above all, the substantial quantity of number concepts, skills and abilities that kindergarten entrants possess can well indicate that mathematics is a part of the preschool child's experience. Young children are surprisingly competent at understanding simple numerical concepts. It is not the case that 6-year-olds enter their first arithmetic class ignorant of the subject matter, but rather, they come with a substantial fund of knowledge.

Conclusions

After reading an extensive review of literature on children's acquisition of numerical concepts, my conclusion is that it is difficult to define how a child really learns. Piaget's stages of intellectual development, as well as his 'assumed' stages of mathematical concept acquisition may

serve as a sort of yardstick in measuring children's learning, but still they are not difinite. Too little is done on young children's method of learning that is difficult to draw much general conclusions based on the limited data.

It seems to me that the knowledge of kindergarteners are so far-ranging that I assume it would be a problem for educators to develop appropriate programs to fit in all these children's needs. In this case, maybe an inventory of the child's number concepts during the first part of the kindergarten year will probably be helpful to achieve what the child has learned already. More emphasis can then be placed in both the kindergarten and the first grade to develop the understanding of the numerical concepts for those children whose background experiences may have created some 'handicapped problems'.

Another point of importance from this review is concerned with teaching numerical concepts to preschoolers. From the studies, it is obvious that preschoolers have quite a rich or substantial knowledge of numerical concepts. However, there are such variations in their acquired knowledge due to their background experiences and environmental factors that individual attention would probably be appropriate. Even though a number of learning theorists have stated that readiness is important for a child's learning, it does not mean that we have to sit back and wait for these young children to be ready. There are things that can be done to encourage children to construct number concept, such as teaching

number concepts to children with concrete objects around their environment that is meaningful to them. It may be a bit early to teach mathematical operations to preschoolers, but they can at least be prepared to understand the meaning of 'adding some' and 'taking away'.

The number concept is really an abstract idea, and is a difficult learning task for young children. However, number concept is the foundation for all mathematical concepts. Therefore, it is important for educators to help the children build a solid groundwork, so that they can be better prepared for all intricate levels of mathematics as they proceed. With the existing number of studies done on young children in this area, there seems to be very limited basic information that educators can retrieve. Probably there is a need for more current research to be done on young children's acquisition of numerical concept at the preoperational stage. We must understand how children learn mathematics before we can really decide on what, when , and how to teach them. After all, a well-planned mathematics program for young children should include all the basics that a child needs.

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